Curve Fitting: Modeling with Quadratic Functions

Goal 1: Write quadratic functions given characteristics of their graphs.
Goal 2: Use technology to find quadratic models for data.

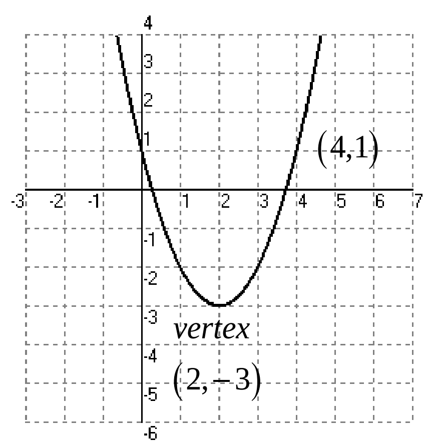
Prior Knowledge: Solving a system of equations using Elimination or Substitution, or with Inverse Matrices methods.

Warm-up:

Solve the system of equations using Elimination or Substitution. $\begin{cases} 2x - y + z = 2 \\ x + y + z = 3 \\ -3x - 2y + z = -4 \end{cases}$

Example 1 **Writing a Quadratic Function in Vertex Form**
Write a quadratic function for the parabola shown below.

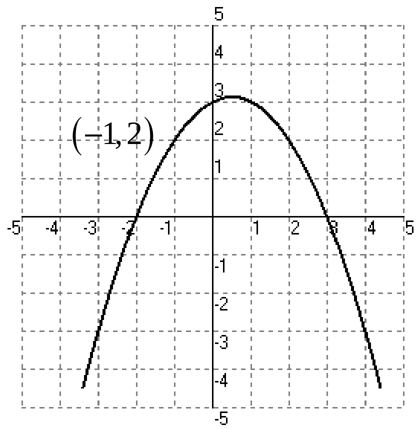
$$y = a(x-h)^2 + k$$



<u>Example 2</u> Writing a Quadratic Function in Intercept Form

Write a quadratic function for the parabola shown at the right.

$$y = a(x-p)(x-q)$$



<u>Example 3</u> Writing a Quadratic Function in Standard Form

Write a quadratic function that fits the points $(0,5)$, $(2,1)$, and $(3,2)$.

(x, y)	$ax^2 + bx + c = y$	System of Equations
$(0,5)$		
$(2,1)$		
$(3,2)$		

Solve the system using Elimination or Substitution methods.

<u>Example 4</u> Writing a Quadratic Function in Standard Form
Write a quadratic function that fits the points $(-2,-5)$, $(1,1)$, and $(3,-15)$.

(x, y)	$ax^2 + bx + c = y$	System of Equations
$(-2,-5)$		
$(1,1)$		
$(3,-15)$		

Solve the system using Elimination or Substitution methods.

<u>Example 5</u> Finding a Quadratic Model for a Data Set

A study compared the speed x (in miles per hour) and the average fuel economy y (in miles per gallon) for cars. The results are shown in the table.
(Source: *Transportation Energy Data Book*)

Speed, x	15	20	25	30	35	40
Fuel Economy, y	22.3	25.5	27.5	29.0	28.8	30.0
Speed, x	45	50	55	60	65	70
Fuel Economy, y	29.9	30.2	30.4	28.8	27.4	25.3

a) Use a graphing calculator to find the best-fitting quadratic model for the data.

1 **$\{x_1, x_2, \ldots\} \rightarrow L_1$; $\{y_1, y_2, \ldots\} \rightarrow L_2$**
Enter the data into two lists on the graphing calculator.

2 **2^{nd} Y= → STATPLOT ON**
Set-up the type of graph to display. Then hit **ZOOM 9**.

3 **STAT → CALC → 5: QuadReg**
Use the quadratic regression feature to find the best-fitting model for the data. Write the a, b, and c values with three significant digits in the answer.

b) Find the speed that maximizes a car's fuel economy.

1 Use the $-\frac{b}{2a}$ rule, and do it algebraically.

<OR>

2 Graph your equation in the **Y=** and go to **2^{nd} TRACE → 4:maximum** to use the MAXIMUM feature on the graphing calculator.