Select Example Problems for AP Calculus Summer Review Work

Linear Equations

You will often be expected to write equations of lines having certain characteristics. Remember that you always need two pieces of information to write any linear equations: a point and a slope.

Example 1 Write the equation in Standard Form for the line through $(2,3)$ and having slope $-\frac{1}{2}$.

$$y-3=\frac{-1}{2}(x-2)$$
$$2(y-3)=-1(x-2)$$
$$2y-6=-x+2$$
$$x+2y=8$$

In this example, you are already given a point (x_1, y_1) and a slope m, so plug them into Point-Slope Form: $y-y_1=m(x-x_1)$. Cross-multiply the denominator of the slope to the left side and then distribute. Finally, Standard Form means variables should be together on one side of the equation and the constant on the other side.

Example 2 Write the equation in Standard Form for the line through $(-5,12)$ and perpendicular to the line $3x-4y=1$.

$$m=-\frac{A}{B}=-\frac{3}{-4}=\frac{3}{4}$$
$$m_{\perp}=-\frac{4}{3} \text{ and } \text{Pt}=(-5,12)$$

In this example you are given the point, but you must determine the slope. Start by finding the slope of the line that is given using the $-\frac{A}{B}$ rule since it is in Standard Form. Since your new line it to be perpendicular, you need the opposite reciprocal of the slope.

$$y-12=\frac{-4}{3}(x+5)$$
$$3y-36=-4x-20$$
$$4x+3y=16$$

Now that you have your perpendicular slope, use it with the given point and it is worked just like example 1 above.

Example 3 Write the equation in Standard Form for the perpendicular bisector of the segment between $(-1,-8)$ and $(-5,12)$.

$$\text{Mpt}=\left(\frac{-1-5}{2},\frac{-8+12}{2}\right)=\left(\frac{-6}{2},\frac{4}{2}\right)=(-3,2)$$
$$m=\frac{12-(-8)}{-5-(-1)}=\frac{20}{-4}=-5 \Rightarrow m_{\perp}=\frac{1}{5}$$

$$y-2=\frac{1}{5}(x+3)$$
$$5y-10=x+3$$
$$-x+5y=13$$

You need to get both your point and your slope in this example. Since your new line is to be the bisector, you want your point to be the midpoint. Get this by averaging your x's and averaging your y's. Get your slope by the "change in y over change in x" formula $m=\frac{y_2-y_1}{x_2-x_1}$, then use the opposite reciprocal since your slope needs to be perpendicular. You have your needed information, so it is again just like example 1 from here.

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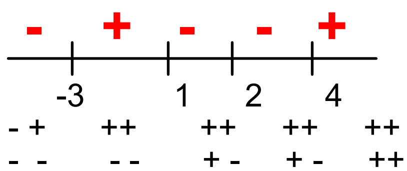
Function Analysis and Sign Charts

The concept of using a sign chart to analyze function attributes is very important in Calculus. In Algebra or Precalculus, you learned to find the zeros of a function and create a sign chart to determine where the function is positive or negative. We will extend this concept in Calculus to also determine where functions are increasing or decreasing, and concave up or concave down.

For now, though, you are expected only to be able to determine where the function is positive, negative, or zero, and also state the domain of the function.

Example 1 $f(x)=\dfrac{(x+3)(x-2)^2}{(x-1)(x-4)}$



Zeros: $x=-3 \; or \; 2$

Domain: $\{x|x\neq 1 \; or \; 4\}$

Positive: $(-3,1)\cup(4,\infty)$

Negative: $(-\infty,-3)\cup(1,2)\cup(2,4)$

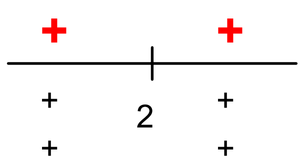
Since this rational function is already in factored form, we know the function equals zero at $x=-3$ and $x=2$, because those are the zeros on top. We also know the function is undefined at $x=1$ and $x=4$, since those values make zero on bottom, and that is not allowed.

To make a sign chart, we place these four values on a numberline and choose "test points" in each interval created. (I used -10, 0, 1.5, 3, and 10 for my test points.) Plug the test point in for x in each factor of the function and write down the sign produced. Decide what the overall sign will be from the combination you wrote down and place this result above the numberline (in red, here). The resulting signs provide the positive/negative interval answers.

Example 2 $f(x)=\dfrac{x-2}{x^3-2x^2+x-2}$

$$f(x)=\frac{x-2}{x^2(x-2)+1(x-2)}$$

$$f(x)=\frac{x-2}{(x-2)(x^2+1)}=\frac{1}{x^2+1}$$



Zeros: None

Domain: $\{x|\; x\neq 2\}$

Positive: $(-\infty,2)\cup(2,\infty)$

Negative: Nowhere

Begin this example by factoring the denominator; since there are four terms, you must factor by grouping. Once it is factored, you may cancel the common $(x-2)$ from top and bottom to reduce the fraction. There are no zeros for this function, since there is no variable left on top (and 1 cannot equal 0). The function is undefined only at $x=2$ since the x^2+1 factor produces only imaginary answers.

In making the sign chart, our only value is $x=2$, so choose test points to either side of 2 and look at the signs produced. The top of the fraction is always positive, and the bottom of the fraction is always positive, so the overall answer is always positive. Note: the point $x=2$, taken out of the domain is called removable discontinuity, which produces a hole in the graph.

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Fractional Exponents

You are expected to be able to factor and simplify expressions containing fractional exponents.
Directions: Factor and express as a quotient with positive exponents.

Example 1 $-3x^{\frac{2}{3}} + x^{\frac{-1}{3}}$

$$= x^{\frac{-1}{3}}\left(-3x+1\right)$$
$$= \frac{\left(-3x+1\right)}{x^{\frac{1}{3}}}$$

Factor our what is common to both terms; it will always be the smallest power that you can factor out, in this case $x^{-\frac{1}{3}}$. Check your exponents to make sure they produce the original expression when added together, in this case the powers on the x's, $-\frac{1}{3}+1$, add to make the original $\frac{2}{3}$. Get rid of the negative exponent by moving that term to the denominator of the fraction. This is the answer since it is a quotient with positive exponents.

Example 2 $2x^{\frac{3}{5}} - \frac{1}{3}x^{\frac{-2}{5}}\left(x-1\right)$

$$= \frac{1}{3}x^{\frac{-2}{5}}\left[6x-\left(x-1\right)\right]$$
$$= \frac{1}{3}x^{\frac{-2}{5}}\left(5x+1\right)$$
$$= \frac{5x+1}{3x^{\frac{2}{5}}}$$

Factor out what is common to both terms; in this example, take out the fraction as well so that it is easier to simplify in the end. To figure out what you get inside the brackets, divide the 2 by $\frac{1}{3}$ (since that is what you're factoring out) and you get 6. Check your exponents to make sure they produce the original expression when added together: $-\frac{2}{5}+1=\frac{3}{5}$. Simplify inside the brackets and get rid of the negative exponent by moving that term to the denominator.

Example 3 $a^{-3}b^{\frac{1}{2}} + 2ab^{\frac{-1}{2}}$

$$= a^{-3}b^{\frac{-1}{2}}\left(b+2a^{4}\right)$$
$$= \frac{b+2a^{4}}{a^{3}b^{\frac{1}{2}}}$$

Factor out what is common to both terms; Check your exponents to make sure they produce the original expression when added together:

Get rid of the negative exponents.

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Composition and Decomposition of Functions

The ability to break down functions into smaller, simpler functions is necessary for computing derivatives and integrals in calculus.

Example 1 Given $h(x) = 2x - 3$ and $g(x) = x^2$, find the following compositions:

A) $h(g(x))$

$= h(x^2)$

$= 2(x^2) - 3 = 2x^2 - 3$

> With compositions, remember to start on the inside and work out from there. Inside the parentheses, replace the $g(x)$ function with its equivalent expression, x^2. Then, in the h function, plug in the x^2 wherever you have an *x*.

B) $g(h(x))$

$= g(2x - 3)$

$= (2x - 3)^2$

$= 4x^2 - 12x + 9$

> Inside the parentheses, replace the $h(x)$ function with its equivalent expression $2x - 3$. Then, in the g function, plug in the $2x - 3$ where you see the *x*, and be sure to use parentheses, too. Finish the problem by FOILing out the expression.

C) $h(h(x))$

$= h(2x - 3)$

$= 2(2x - 3) - 3$

$= 4x - 9$

> Replace the x in the h function with the entire h function, distribute, and simplify.

Example 2 Decompose $k(x) = \sin^5(x^2 + 1)$ into three separate functions.

Let $g(x) = x^2 + 1$

$h(x) = \sin(x)$

$f(x) = x^5$, then

$k(x) = f(h(g(x)))$

> Start on the inside of the $k(x)$ function and "build it." Since the inside is $x^2 + 1$, let this be the first function we define. The next thing that is applied to this expression is the sine function, so that becomes our second function we define. And finally, the sine function has a 5^{th} power applied to it, so x^5 becomes our third function defined. When you plug each part into the next, you will build the original $k(x)$ function.